Research Highlight: Hyperbolic jigsaws and families of pseudomodular groups

Work of Professor Tan Ser Peow

The cusp set of the modular group PSL(2,Z) is the set of rationals including infinity. This follows from the Euclidean algorithm. In general, determining the cusp set of a discrete subgroup of PSL(2,R) is a difficult question and few examples are known where the cusp set is completely determined.

Long and Reid defined a pseudo-modular group to be a Fuchsian group which is NOT commensurable with the modular group, but which has the same cusp set (commensurable groups have the same cusp set). In a paper in Crelle in 2002, they constructed a very small number of examples of such groups and asked if there were infinitely many commensurability classes of pseudo-modular groups. Very little progress was made on this question until recently when Prof Tan Ser Peow together with two undergraduate students Lou Beicheng and Vo Anh Duc working with him on UROPS projects succeeded in showing that there were indeed infinitely many commensurability classes of pseudo-modular groups.

They did this by introducing a novel construction of surfaces whose fundamental domains are obtained by gluing together marked ideal triangular tiles, which they call hyperbolic jigsaw surfaces. In the case of jigsaw surfaces made up of the two simplest types of tiles, they were able to show that indeed all the groups associated to these surfaces were pseudo-modular groups and that furthermore, they belong to infinitely many different commensurability classes.

References:

D.D. Long and A.W. Reid. Pseudomodular surfaces, J. Reine Angew. Math. 552, (2002), 77--100.

Beicheng Lou, Ser Peow Tan, and Anh Duc Vo. Hyperbolic jigsaws and families of pseudomodular groups, I. Geometry & Topology 22, (2018) 2339-2366.