

## **Research Highlight: A long chain of P-points**

### **Work of**

**Associate Professor Dilip RAGHAVAN (NUS) and**

**Dr Borisa KUZELJEVIC (University of Novi Sad)**

A P-point is a special type of ultrafilter on the natural numbers that has a fine ability to detect convergence behaviors. P-points were introduced by W. Rudin [3] in 1956, who proved their existence under the Continuum Hypothesis (CH).

The Rudin--Keisler ordering of ultrafilters is a way to compare ultrafilters in terms of their ability to detect convergence behaviors. Under CH, the class of P-points has a rich structure with respect to this ordering, although many questions remain.

In 1973, Blass [1] asked which ordinals are embedded into the class of P-points with respect to the Rudin--Keisler ordering assuming CH. In [2] we showed that the successor of the continuum can be embedded. This completely answers Blass' question because every ultrafilter has at most continuum many predecessors. So the ordinal which is one more than the the successor of the continuum cannot be embedded.

### **Reference**

1. Blass, A., The Rudin-Keisler ordering of P-points, Transactions of the American Mathematical Society, Vol. 179, 1973, pp. 145--166.
2. Kuzeljevic, B. and Raghavan, D. A long chain of P-points, Journal of Mathematical Logic, vol. 18, no. 1, 2018, 38 pp.
3. Rudin, W., Homogeneity problems in the theory of Czech compactifications, Duke Mathematical Journal, vol. 23, 1956, pp. 409--419.