Research Highlight: The mathematics of measuring financial risks

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An important problem in finance is how to measure the monetary risk associated with a financial portfolio. In [1], Artzner et al proposed a set of mathematical axioms for a risk measure, which was subsequently enlarged upon by Foellmer and Scheid [3]. In these approaches, financial positions are modelled as random variables on a probability measure space. A *risk measure* is an extended real valued function □ so that for each random variable X (in some model space), □(X) can be interpreted as a monetary measure of the risk involved in the financial position X.

Arising from considerations of computational effectiveness and robustness of a risk measure, it is deemed desirable for a risk measure to be representable via its dual functional (in the sense of convex analysis). Such a representation was first established by Delbaen [2] when the model space consists of all bounded measureable functions. Subsequent investigations [4], however, showed that dual representations of risk measures are not always achievable when the model space contains unbounded positions. In practical situations, risk measures are often *law-invariant*, i.e. financial positions that are equal in distribution carry the same risk. Gao, Leung, Munari and Xanthos [5] showed that law-invariant risks measures with the Fatou property defined on Orlicz spaces are always dual representable. This result was later extended by Tantrawan (PhD student in the Department) and Leung [6] to all law-invariant risk measures defined on any rearrangement invariant space.

References:

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[5] N. Gao, D. Leung, C. Munari and F. Xanthos, Fatou property, representations, and extensions of law-invariant risk measures on general Orlicz spaces, Finance and Stochastics 22, 2018, 395-415.

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