4TH NUS GRADUATE SYMPOSIUM IN MATHEMATICS

18 April 2016 (Monday)
Department of Mathematics
S17 #04-06 & #04-05

Faculty of Science
Department of Mathematics

NUS GRADUATE SYMPOSIUM IN MATHEMATICS
18 April 2016, Department of Mathematics, NUS

PROGRAMME

Time/Venue	S17-04-06		S17-04-05	
08:50 – 09:00	**Opening Address** Prof ZHU Chengbo			
09:00 – 09:45	**Twisted Bhargava Cubes and Boxes** Prof GAN Wee Teck	*p2*		
09:45 – 10:15	*Tea break @ Mathematics Department Lounge*			
10:15 – 11:00	**How to construct a large scale optimal valid correlation matrix?** Prof SUN Defeng	*p2*		
11:00 – 11:20	CHEN Bo	*p2*	HU Fei	*p4*
11:20 – 11:40	Andreas GUNAWAN	*p3*	FENG Xianzhe	*p3*
11:40 – 12:00	GUO Han	*p3*	PENG Cheng	*p7*
12:00 – 14:00	*Lunch @ Mathematics Department Lounge*			
14:00 – 14:20	GUO Jiancang	*p3*	LIU Yong	*p5*
14:20 – 14:40	LI Yunzhi	*p5*	LU Hengfei	*p6*
14:40 – 15:00	RUAN Xinran	*p7*	LUO Caihua	*p6*
15:00 – 15:20	LIU Zhaoqiang	*p6*	MENG Sheng	*p6*
15:20 – 15:50	*Tea break @ Mathematics Department Lounge*			
15:50 – 16:10	XU Guodong	*p8*	REN Shiquan	*p7*
16:10 – 16:30	HUANG Shan	*p4*	SONG Xu	*p7*
16:30 – 16:50	JIANG Wei	*p5*	WEI Ran	*p7*
16:50 – 17:10	ZHAO Quan	*p8*	ZENG Yishu	*p8*

(Number in italic denotes page number of abstract)

ABSTRACTS

Prof GAN Wee Teck
Twisted Bhargava Cubes and Boxes

In his groundbreaking thesis work from 2001, Manjul Bhargava extended Gauss's composition laws for binary quadratic forms to higher degree forms. One crucial ingredient in his work is the parametrisation of the orbits of lattice points in a prehomogeneous vector space by quantities of arithmetic interest. Using the SL(2,Z)x SL(2,Z)xSL(2,Z)-action on 2x2x2 cubes, he gave a simpler description of Gauss's law, and using the SL(2,Z) x SL(3,Z) x SL(3,Z)-action on 2x3x3 boxes, he discovered a composition law on binary cubic forms. In this lecture, we revisit these two orbit problems over fields (rather than integers) and consider twisted versions of them.

In particular, we shall discuss the question: what are the orbits parametrised by?

Prof SUN Defeng
How to construct a large scale optimal valid correlation matrix?

In finance, risk management, and many other areas, one often has to deal with invalid correlation matrices. Mathematically, a given symmetric matrix is an invalid correlation matrix if and only if its smallest eigenvalue is negative assuming that all its diagonal entries are ones. The question is how to construct a reasonably good correlation matrix from the given invalid one. Statistically, one could argue that such an invalid correlation matrix is not properly formulated and one should use better statistical methods to reconstruct a valid one. However, in practice, it is more often than not that the reformulated correlation matrix contains quite a number of negative eigen-values albeit of small magnitude. There are a number of reasons contributing to this phenomenon: insufficient/ missing raw data, non-synchronous data, human factors, and so on. In this talk we aim to construct an optimal valid correlation matrix of dimensions up to 10,000 by 10,000 from the observed one by using modern non-smooth optimization theory, in particular, on the second order sparsity of the metric projector over the cone of symmetric and positive semi-definite matrices. Computer codes in Matlab/R/Python for solving the correlation matrix problems will be made available to all the participants.

CHEN Bo
An Introduction to Linear Elliptic Control Problems with L^1 Cost Functional

In this talk, the linear elliptic equation constraint sparse optimal control problem is considered. Firstly we introduce how we discretize the infinite dimensional optimization problem using the finite element method. Secondly, we present several methods to solve the discretized finite dimensional optimization problem. Besides directly using the Newton's method, we also apply the ABCD method for its dual problem.

FENG Xianzhe
A note on non-linear biseparating operators

In my talk, I will introduce the history and motivation of the study of biseparating operator. I will also talk about the characterization of non-linear biseparating operator from C(X) to C(Y), where X, Y are compact Hausdorff spaces or realcompact spaces. The relationship with order isomorphism will also be discussed.

Andreas Dwi Maryanto GUNAWAN
Verifying a Network Model for Evolution

Tree Containment Problem (TCP) is a problem in phylogenetics that arises for the purpose of model verification. A lot of effort has been made to find some classes of networks, on which the TCP can be solved in polynomial time. Such network class has the potential to be used for modelling evolution of life. In this talk, I will briefly explain the TCP, give some motivation that makes TCP interesting, and finally show some recent results on the topics.

GUO Han
Kurdyka- Lojasiewicz inequality and error bounds

We study the Kurdyka- Lojasiewicz (KL) exponent, an important quantity for analyzing the convergence rate of first-order methods. Specifically, we develop various calculus rules to deduce the KL exponent of new (possibly nonconvex and nonsmooth) functions formed from functions with known KL exponents. In addition, we show that the well-studied Luo-Tseng error bound together with a mild assumption on the separation of stationary values implies that the KL exponent is 1/2. Moreover, we explore the interplay between error bounds and the Kurdyka-Lojasiewicz (KL) inequality for convex functions. One can show the equivalence between the two concepts for convex functions having a moderately flat profile near the set of minimizers.

GUO Jiancang
An Introduction of Climbing String Method in Collective Variables

It is frequently the case that the progress of some chemical, mechanical, or thermodynamics process can be followed by following the evolution of a small subset of generalized coordinates in a system. When generalized coordinates are used in this manner, they are typically referred to as reaction coordinates, collective variables, or order parameters. A brief introduction of climbing string method in collective variables, which was proposed to compute the saddle points and the corresponding MEPs for a given minimum of the potential or free energy of complex systems. These saddle points act as transition states for the barrier-crossing event. Application to alanine dipeptide and Deca-alanine will be presented.

HU Fei
The Lüroth problem

In the nineteeth century, Lüroth proved that every non-trivial subfield L of $k(x)$, the rational function field with only one indeterminate x, is also rational, i.e., $L=k(f)$ for some rational function f. Geometrically, this states that if there is a non-trivial dominant rational map from the projective line $\mathbb{P}^1$ to a curve C, then C is rational (birational to $\mathbb{P}^1$).

Lüroth asked whether the similar question is still true in any dimension, now this is known as the Lüroth problem. Geometrically, it can be restated as:
Is every unirational variety rational?

In dimension two, this question was affirmatively answered by Castelnuovo using his rationality criterion in 1894.

In dimension three, this question was a longstanding conjecture until 1972 - three quite different counter-examples appeared: Clemens-Griffiths, Iskovskikh-Manin, and Artin-Mumford.

I would like to give a brief review of these three different type methods.

HUANG Shan
Opaque Assets and Optimal Bank Capital

Banks' assets are opaque and, therefore, we model their accounting asset values as partially observed variables that have static uncertainty terms. We derive a stochastic control model for this situation and calibrate that to a sample of U.S. banks. Under the calibrated model, the banks have incentive to add noise to the reported accounting asset values and smooth their asset values to hide their solvency risk from banking regulators. The partially observed model explains the banks' actions significantly better than the corresponding fully observed model, indicating that the banks consider the accounting noise and its benefits when optimizing their equity capital level. Further, due to the substantial shock on the asset values, the banks' assets were more opaque during the recent financial crisis than outside that.

JIANG Wei
Simulating Risk Measures

Risk measures, such as value-at-risk and expected shortfall, are widely used in risk management, as exemplified in the Basel Accords proposed by Bank of International Settlements. We propose a simple general framework, allowing dependent samples, to compute these risk measures via simulation. The framework consists of two steps: in the S-step, risk measure is estimated by using selected sorting algorithm; in R-step, necessary sample size is computed based on newly derived asymptotic expansions of relative error for dependent samples, and the S-step is repeated until requirement on relative error is met. We systematically investigate various sorting methods in the S-step. Numerical experiments indicate that the algorithm is easy to implement and fast, compared to existing methods, even at the 0.001 quantile level. We also give a comparison of the relative errors of value-at-risk and expected shortfall.

Keywords
Relative error, importance sampling, _-mixing, order statistics, estimation time.

LI Yunzhi
Numerical Study of Vapor Condensation and Wetting Transition on Patterned Surface using String Method

In the first part, we study vapor condensation on hydrophobic surfaces patterned with microscale pillars. The critical nuclei, the activation barriers, and the minimum energy paths are computed using the climbing string method.

In the second part, we study wetting transition on hydrophobic grooved surface using molecular dynamics. We use density field of particles as the collective variables. Then we apply climbing string method to find out the transition state in the space of collective variables.

LIU Yong
Introduction to reverse mathematics

This talk is an introduction to reverse mathematics, with an emphasis on the principles ADS and CAC. ADS states that every infinite linear order has an ascending or descending sequence, and CAC states that any infinite partial order has an infinite chain or anti-chain.

LIU Zhaoqiang
Tensor Decompositions for Learning Latent Variable Models

I will mainly talk about the corresponding paper. This paper considers a computationally and statistically efficient parameter estimation method for a wide class of latent variable models—including Gaussian mixture models, hidden Markov models, and latent Dirichlet allocation—which exploits a certain tensor structure in their low-order observable moments (typically, of second- and third-order). Specifically, parameter estimation is reduced to the problem of extracting a certain (orthogonal) decomposition of a symmetric tensor derived from the moments; this decomposition can be viewed as a natural generalization of the singular value decomposition for matrices. Although tensor decompositions are generally intractable to compute, the decomposition of these specially structured tensors can be efficiently obtained by a variety of approaches, including power iterations and maximization approaches (similar to the case of matrices).

LU Hengfei
Global GSp(4)-distinguished cuspidal automorphic form

Assume E/F is a quadratic number field extension. With the help of the extended global theta lift and Eisenstein series, we will use the regularized Siegel Weil formular and orbit decomposition to deduce the partial results of generic cuspidal automorphic form on GSp(4) related to the periods over the quadratic extension E/F.

LUO Caihua
From modular forms to Langlands program

Around 1967, Langlands formulated a far-reaching program, the so-called Langlands program, which is a synthesis of several important themes in classical number theory. At the heart of Langlands' program is the general notion of automorphic representations and its L-functions.

Starting from classical modular forms on SL(2,R), we will take a trip to reformulate the classical theory under modern automorphic forms framework which plays a key role in the Langlands program.

MENG Sheng
Fixed Point Problem in Algebraic Geometry

We shall introduce the almost fixed point problem in algebraic geometry and show some recent work.

PENG Cheng
On the Ramsey's Theorem for Pairs

This talk will give an introduction to the reverse mathematics and related parts of computability theory. We will focus on Ramsey's theorem as a case in reverse mathematics analysis and we will introduce Ramsey's theorem for pairs, and some fundamental tools such as cohesiveness and Mathias forcing.

REN Shiquan
Applications of configuration spaces of manifolds

In this talk, we study two applications of configuration spaces of manifolds. One application is to study the existence problem of k-regular maps on manifolds. And the other application is to study the order of vector bundles over configuration spaces of manifolds.

RUAN Xinran
Fundamental Gaps and Energy Asymptotics of the Gross-Pitaevskii/nonlinear Schrodinger Equation with Repulsive Interaction

We study asymptotically and numerically fundamental gaps (i.e. difference between the first excited state and the ground state) in energy and chemical potential of some specially defined Gross-Pitaevskii equations (GPE) with repulsive interaction under box potential or harmonic potential. And some gap conjectures will be formulated based on our results. Results for GPE on bounded domains with either homogeneous Neumann BC or periodic BC will also be provided.

SONG Xu
From theta correspondence to (τ,b)-theory

First we will briefly recall some basic facts and important results in theta correspondence. Then we will introduce the (τ,b)-theory which can be regarded as a generalization of theta correspondence and automorphic descent.

WEI Ran
The High-Temperature Behavior for the Long-Range Directed Polymer with Marginal Stable Exponent α=1

We study the high-temperature behavior of the free energy of the long-range directed polymer in marginal case, i.e. the stable exponent α=1. We expect to show that the difference p(β) between quenched free energy and annealed free energy is always negative for small β if and only if the random walk S (which represents the polymer chain), whose one step distribution is in the domain of attraction of the Cauchy law, is recurrent.

XU Guodong
Exact Clustering by Convex Optimization with Minimal Separation Condition

In this talk, I will presented a weighted sum-of-l_1-norm based regularization model for convex clustering. Particularly, we rigorously proved that the proposed model always correctly identify the membership of each sample of the data set with two clusters, as long as the interclass distance of two clusters is larger than their inner-class distances. Such a cluster separation requirement for guaranteeing exact clustering is also necessary for many existing heuristic or non-convex clustering methods, and is much weaker than the one established very recently for the sum-of-l_2-norm based regularization method. The proposed model and theoretical analysis provide good insights and solid foundations for the development of more powerful convex clustering methods.

ZENG Yishu
Stationary Makov perfect equilibria in large stochastic games

This paper provides a necessary and sufficient condition for the existence of stationary Markov perfect equilibria in large stochastic games. This work offers a new method, and extends Chakrabarti (2003), who considered the case of finite dimensional action space. Based on the negligible effects of unilateral deviations on transition probabilities, we derive an equivalence theorem, which reduces the original problem to finding a Nash equilibrium in its component game. We borrow technical insights from Mertens (2003), who provided a measurable version of the measurable choice theorem in n-dimensional Euclidean space, and generalize his result to a compact metric space. We prove that nowhere equivalence is necessary and sufficient for the existence of stationary Markov perfect equilibria. This work also explains the connection between finite and large stochastic games by a limit principle, and provides a complete characterisation for its validity in general.

ZHAO Quan
A Parametric Finite Element Method for Simulating Solid-State Dewetting Problems

We propose an efficient and accurate parametric finite element method (PFEM) for solving the sharp interface model of solid-state dewetting of thin films with anisotropic surface energies. The governing equations of the sharp interface model belong to high-order geometric PDEs, which include anisotropic surface diffusion flow and contact line migration. Compared to the traditional methods, the proposed PFEM not only has good accuracy, but also poses very mild restrictions on the numerical stability.