Lie Theory Day

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Programme

9:00am – 10:00am	**Local Descent and Algebraic Wavefront Set** *Lei Zhang* *NUS*
10:00am – 10:30am	Break @ Mathematics Department Lounge
10:30am – 11:30am	**Signatures of invariant Hermitian forms on finite-dimensional representations** *David Vogan* *MIT*
11:30am - 2:00pm	Lunch
2:00pm – 3:00pm	**Developments in the Kazhdan-Lusztig theory of p-adic GL_n** *Maxim Gurevich* *NUS*
3:00pm – 4:00pm	**Period Relations of Standard L-functions of Symplectic Type** *Fangyang Tian* *NUS*
4:00pm – 4:30pm	Break @ Mathematics Department Lounge
4:30pm – 5:30pm	**Special unipotent representations of real classical groups** *Chen-bo Zhu* *NUS*

All talks are held in S17, Room 04-06 (level 4, Colloquium Room)

For directions to S17, refer to http://ww1.math.nus.edu.sg/contactus.aspx

Abstracts

Local Descent and Algebraic Wavefront Set
Lei Zhang

In this talk, we will discuss the local descents for the classical groups of Hermitian type over local fields of characteristic zero. By using the results on local Gan-Gross-Prasad conjecture, we may obtain explicit spectral decomposition of the local descents at the first occurrence index in terms of the local Langlands parameters after explicating calculations of the relevant local root numbers. Conjecturally, this first occurrence index of irreducible admissible representations in generic local Vogan packets equals the largest part of the partition in their algebraic wavefront set.

This is a joint work with Dihua Jiang and Dongwen Liu.

Signatures of invariant Hermitian forms on finite-dimensional representations
David Vogan

Suppose G is a real reductive algebraic group, and π is an irreducible complex representation of G. It often happens that π admits a non-zero G-invariant Hermitian form $\langle\cdot,\cdot\rangle_\pi$. Schur's lemma guarantees that the form is nondegenerate and unique up to a real scalar; so Sylvester's theorem says that the only possible signatures are (p,q) and (q,p). Write $\text{Sig}(\pi) = |p-q|$; the smallness of $\text{Sig}(\pi)$ measures how thoroughly indefinite the form is.

The Weyl dimension formula says that $\dim(\pi)$ is a polynomial of degree equal to $(\dim G - \text{rank}(G))/2$ in the highest weight. I'll prove that $\text{Sig}(\pi)$ is a quasipolynomial of degree $(\dim K - \text{rank}(K))/2$ in the highest weight, with K a maximal compact subgroup of G. This says (for noncompact G) that the signature is ``much smaller'' than the dimension, meaning that the form is very indefinite.

This is joint work with MIT undergraduate Christopher Xu and his grad student mentor Daniil Kalinov.

Developments in the Kazhdan-Lusztig theory of p-adic GL_n
Maxim Gurevich

Decades ago, the theory of Kazhdan-Lusztig polynomials has revolutionized Lie theory, by interpreting key decomposition numbers in representation categories in terms of singularities of certain complex varieties on one hand, while producing hands-on algorithms for computing these numbers on the other hand. Zelevinksi rightly conjectured that the representation theory of GL_n over a p-adic field has a lot to gain from the KL theory.

I would like to survey the consequences of these relations and the developments that followed, concluding with some recent results (which also involve the parabolic analogues of the polynomials). A central theme is that the passage from the value at 1 of a KL polynomial to the full polynomial is equivalent to a quantization of representation-theoretic situations.

Time permits, we will discuss the phenomenon of representations with an irreducible socle.

Period Relations of Standard L-functions of Symplectic Type
Fangyang Tian

In this talk, I will discuss my recent work joint with Dihua Jiang and Binyong Sun on the global period relation for the twisted standard L-function $L(s, \Pi\otimes\chi)$, where Π is an irreducible cuspidal automorphic representation of $GL_{2n}(\mahtbb{A})$ which is regular algebraic and of symplectic type. As far as we know, this is the first unconditional result of this type for higher degree L-functions. Our result generalizes many predecessors' work, such as Ash-Ginzburg, Grobner-Raghuram, Januszewski, to mention a few.

Along this talk, I will also discuss the key ingredients - Non-vanishing hypothesis at archimedean local place and existence of uniform cohomological test vectors, which were essential obstructions in attacking the period relation problem.

Special unipotent representations of real classical groups
Chen-bo Zhu

A fundamental problem in representation theory is to determine the unitary dual of a given Lie group G, namely the set of equivalent classes of irreducible unitary representations of G. A principal idea, originated in a ground-breaking paper of Kirillov, is that there is a close connection between irreducible unitary representations of G and its coadjoint orbits. This is known as the orbit method.

Among other things, the orbit method predicts the existence of (yet to be defined) unipotent representations, those which are expected to be attached to nilpotent coadjoint orbits. Following Arthur, Vogan-Barbasch has given a precise definition of special unipotent representations, which are attached to nilpotent orbits of $G^{\vee}$ (the Langlands dual of G).

In a joint work with J.-J. Ma and B. Sun, we construct all special unipotent representations of a real classical group G attached to quasi-distinguished nilpotent orbits of $G^{\vee}$, by using the method of theta lifting. We also show the unitarity of these representations. The construction of all special unipotent representations for such a G is work in progress (joint with Barbasch, Ma and Sun).