Research Breakthrough

MATHEMATICS

Nonlinear Wave Propagations Over a Boltzmann Shock Profile

Professor Yu Shih-Hsien

This is a summary of the work of Professor Yu Shih-Hsien, in [Y]. This work completes the studies on shock profile stability problem initiated in 1985.

A shock wave is a generic phenomenon for viscid compressible fluids such as ideal gases modeled by the compressible Euler equations. It is a compressive discontinuity occurring in a fluid. Its presence within a fluid makes the physics for the compressible fluid and the related mathematical theories highly nontrivial and interesting.

The time asymptotic stability of a viscous shock profile was obtained independently by Goodman and Matsumara by an energy method under a zero total mass condition in 1985. These two works initiated sequential works in this direction, to remove the zero total mass condition, as well as to construct more detailed structure of the perturbations so that the results could be applied to the future studies on the vanishing viscosity problem.

Professor Yu's main idea in studying the qualitative time asymptotic stability of a shock layer is based on a separation of scales. The separation is based on the nature of the hyperbolic wave propagation around a shock wave that has a rather simple wave structure. It gives a natural wave to decompose a vector into two components, one tending to propagate to the far fields while the other tending to merge into the shock front. The propagating trajectories of the components are the characteristic curves of the hyperbolic system. Then, one needs to realize this phenomenon in terms of viscous conservation laws or Boltzmann Equation.

The structure of the wave propagation towards far fields in both Boltzmann Equation and viscous conservation laws can be identified with the wave structures of the Green's Function of the problems linearized around their far fields. Through a T-operator which solves the initial value problem and a C-operator which constructs the coupling due to the zero total mass input function, Prof Yu solved the shock profile stability problem. This work has been accepted for publication in the *Journal of American Mathematics Society*.

[Y] Yu, S-H. Nonlinear Wave Propagations over a Boltzmann Shock Profile, *J.Amer. Math. Soc.*, No. 4, Vol 23, (2010) 1041-1118

INTERVIEW with Professor Yu Shih-Hsien

http://ww1.math.nus.edu.sg/onepageCV.aspx?id=matysh

'Romancing the Fundamentals to Build a Monument'

Prof Yu Shi-Hsien's journey has taken him from National Taiwan University, where he did his Master's degree, to Stanford University for his Ph.D. Later, he joined UCLA, then Osaka University, followed by City University of Hong Kong, and finally arriving at NUS in December 2007 to continue his adventure in mathematical fundamental research. In this interview, he shares some of the exciting moments in his career.

Tell us about your research - background, discoveries and future plans.

My work is in basic research to lay down the fundamental principles of Physics. The research is primarily on gas dynamics, in particular on rarefield gas. For the past 12 years, I have been working on the problem of shock profile stability, which has been around since the 80's. Finally, everything has come together. Now I have developed a unified proof for the stability problems of shock profiles for viscous conservation laws and Boltzmann Equation.

The turning point came during the SARS outbreak when I was working in City University of Hong Kong. At that time, I was stuck at a critical point. I was on my own and spent a lot of time pacing up and down the length of Victoria Harbour. One day, I found the solution! From the breakthrough, I continued to develop it to get the final solution. It has been accepted by a journal and will be published in December this year.

When I was in America in 1997, I solved the problem of the discrete shock wave related to the KAM theorem – Kolmogorov Arnold Moser. This was unique in that I set out to prove wrong thing, but ended up discovering something more complex and more beautiful.

Fundamental research has no 'next stage'. I will work on other problems that have fascinated me during my journey. I love doing this - it is an adventure that always takes you by surprise. I may set out in one direction but later find myself ending up in a completely different place. Like Columbus, who set sail for India but discovered the Americas instead.

What inspires you in your research?

I am relaxed and I enjoy my research. If I make a mistake, it is okay. Mistakes are creative! They point me in new directions. I am constantly romancing the fundamentals to build a monument.

Prof Yu is a mathematician who exudes the spirit of an artist.