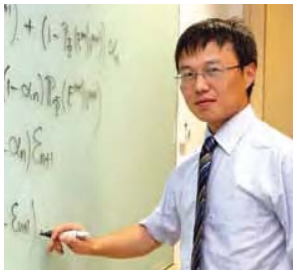
RESEARCH

# Department of Mathematics (cont'd)

## Research Breakthrough
## Typical Dynamics of Uni-Critical Polynomials

**Prof Shen Weixiao**
*http://ww1.math.nus.edu.sg/staffCV/matsw.htm*
**Department of Mathematics**



A guiding goal of the modern theory of dynamical systems is to look for a satisfactory description of most such systems.

Prof Shen Weixio's recent work examines the iteration of uni-critical polynomials as dynamical systems acting on both the real line and the complex plane. He and a few collaborators have proven a trichotomy for almost every uni-critical polynomial.

Their findings show that for almost every real or complex parameter, the corresponding dynamical system is either uniformly hyperbolic, infinitely renormalisable or allows a satisfactory statistical description.

This work is based on two recent results found for non-infinitely-renormalisable uni-critical polynomials: the proving of a lower bound for the moduli of a sequence of dynamically defined annuli[1], and evidence that shows two combinatorially close uni-critical polynomials are actually close[2].

In their most recent work, Prof Shen and his collaborators transferred these estimates to the parameter space by studying the analytic properties of 'holomorphic motion'.

The findings extend our understanding of the dynamics of uni-critical polynomials to the same level as quadratic polynomials.

***Reference:***

1. Kahn, J., Lyubich, M.: Local connectivity of Julia sets for unicritical polynomials. *Ann. of Math*. 170, 413–426 (2009)

2. Avila, A., Kahn, J., Lyubich, M., Shen, W.: Combinatorial rigidity for unicritical polynomials. *Ann. of Math*. 170, 783–797 (2009)



# INTERVIEW with
## Prof Shen Weixiao

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Prof Shen Weixiao received his BSc from the University of Science and Technology of China in 1995 and furthered his studies with an MSc in 1998 and a PhD in 2001, both from the University of Tokyo. He was a research fellow at the University of Warwick until 2004, after which he returned to the University of Science and Technology of China as Professor. He joined NUS' Department of Mathematics in 2009 and has been directing his research focus on dynamical systems.

In this interview, Prof Shen describes his long-standing interest in those systems and his recent work on uni-critical polynomials.

***What led you to the study of dynamical systems?***

I have been interested in mathematics since childhood and chose dynamical systems as the focus of my graduate study for two reasons. First, the theory of dynamical systems is related to various fields of science. In principle, the goal is to study the evolution of all systems in the world, although the current technique only allows us to understand little about the chaotic behaviour of most systems. Second, knowledge in other sub-field of mathematics, such as analysis, topology and geometry, has potential applications in the theory of dynamical systems.

***Within that context, how would you define a satisfactory research result?***

I would say that a satisfactory research result is one that solves a natural problem in a natural manner. A problem is considered natural if it arises from the real world or, it is in need of a theory, in the case of pure mathematics. A method is natural if it identifies the difficulties of the problem and clearly shows its merits and limitations.

***Why are uni-critical polynomials so important?***

General dynamical systems have complicated behaviour and a big part of the world is still mysterious. Uni-critical polynomials are the simplest non-linear dynamical systems. These systems already display many interesting forms of chaotic behaviour but are approachable from a mathematical point of view. There are many techniques from complex analysis and hyperbolic geometry that are applicable to the study of these systems.

***What difficulties did you face in moving away from the use of quadratic polynominals?***

There was a particular technical difficulty. Quadratic polynomials have a special property – the linear growth of principal moduli – that provides perfect control of distortion in the systems, although it fails for higher degree uni-critical polynomials. We had to show that the known distortion control is enough for there to be dynamical consequences.

***How will your work affect the wider field of mathematics?***

Researchers focusing on dynamical systems are looking for a guiding law that holds for almost every system. Our work verifies a conjectured law in the special setting of uni-critical polynomials.