Cover Story
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Numerical Methods *for* Blind Image De-convolution

Developing New Mathematical Models and Techniques for Recovering Motion-Blurred Photographs

by Ji Hui

Despite all the advances in digital photography, motion blurring is still one of the most common causes of blurred pictures caused by camera movement during exposure time. A motion-blurred picture is usually modeled by

$$f = p * g + n,$$

where * denotes the discrete convolution operator, f denotes the blurry picture, g denotes the sharp one, n denotes image noise, and p denotes the blur kernel determined by the relative motion between the camera and the scene. Modern cameras address this issue by using image stabilization, which controls mechanical actuators that shift the sensor or lens element during the exposure to compensate for motion of the camera and thereby reduce mage degradation. Unfortunately, image stabilization only addresses very modest camera shake.

Motion de-blurring is about how to recover sharp image g from a blurred image f. Since the camera motion can be arbitrary, the blur kernel p cannot be determined by some pre-process. In mathematical term, such a de-blurring process is often called "blind de-convolution", which simultaneously estimates both the sharp image g and the blur kernel p from a blurry image f. Blind image de-convolution is a challenging ill-posed bi-linear inverse problem. Clearly, there exist an infinite number of solutions. One obnoxious solution is $\{p := \delta; g := f\}$, where δ is the so-called Dirac operator. The result corresponding to such a trivial solution essentially is doing nothing on the given blurred image f. Thus, one fundamental question to answer in blind image de-blurring is how to mathematically characterize the so-called "sharp image". In a variational approach, such a question becomes how to design a function such that it achieves minimum value at a clear image with sharp edges. In recent years, the author and his collaborators have developed a framework for resolving this challenging problem, together with several new mathematical models and computational techniques. The basic idea is to model both blur kernel and sharp images as some signals that are compressible under some transform domain.

A signal g is compressible in some transform means that there exists a system such that the signal g can be described by the linear combination of only a few atoms of this system. In other words, the signal g can be sparsely approximated by the system:

$$g = \sum_n D_j c_j$$

where $W = \{D_j\}$ denotes the system and $c = \{c_j\}$ denotes a sparse coefficient vector with most elements being zeros or close to zeros. Then, given a signal g and its sparsifying system W, one may estimate its associated sparse coefficient vector c via solving an ℓ_1-norm relating optimization problem [2]:

$$\text{argmin}_{\hat{g}} \ \|\hat{g} - g\|_2^2 + \lambda \|c\|_1, \quad \text{such that} \quad \hat{g} = Wc.$$

The system W usually is a redundant system as it provides higher sparse degree of c. One often-used system for sparsifying images is the so-called wavelet tight frame [1]. A wavelet system refers to a system generated by the translates and dilations of a few mother wavelet functions:

$$W = \{\psi_j(2^k \cdot - \ell)\}, \quad j = 1, \ldots, L.$$

The wavelet tight frame is the extension of an orthonormal basis to the case of redundant system, which keeps the so-called perfect reconstruction property:

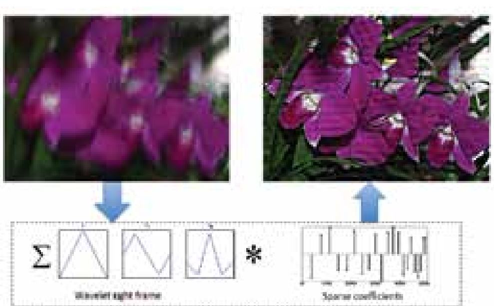
$$g = \sum_{j,k,\ell} \langle g, \psi_j(2^k - \ell) \rangle \psi_j(2^k - \ell), \text{ for all } g$$

Interested readers are referred to [1] for more details.

Built upon the concepts of wavelet tight frames and sparse approximation via ℓ_1-norm relating optimization, the author and his collaborators developed in [3] a powerful sparse approximation based framework for blind image de-blurring:

$$\frac{1}{2}\|p * g - f\|_2^2 + \lambda_1 \Phi(p) + \lambda_2 \Psi(g),$$

where Φ denotes the sparse approximation based regularization for the kernel *p*, and Ψ denotes the sparse approximation based regularization for the sharp image *g*. The optimization above can be solved via the alternating iteration scheme, which alternatingly estimates the kernel *p* and the sharp image *g* via solving an ℓ_1-norm relating optimization problem. In [2], the curvelet is used for sparsely approximating the kernel owing to its curvy support, and wavelet tight frame is used for sparsely approximating the image. In a sequent work [4], the image and the kernel are both sparsely approximated by the wavelet tight frame. In addition, a new analysis sparse model (e.g. $\Phi(p)=\|W^T p\|_1$) is used for better performance. The approach developed in [3, 4] can effectively restore a large class of blurred images owing to camera shake. However, it is observed in [5] that the result obtained via using the ℓ_1-norm based sparse approximation is biased toward slightly blurred result. In other words, the results might still look slightly blurry. Thus, a new sparsity-based prior is proposed in [4] which considers the ratio ℓ_1/ℓ_2 as the energy function for characterizing images with sharp edges.



The illustration of blind image de-blurring via sparse approximation.

The wavelet tight frame-based blind image de-convolution methods [3,4] can effectively remove motion blurring from photography when the blurring effect of the input is uniform over the whole image. However, such an assumption may break down in certain configurations. For example, when the camera shake is dominated by rotation or the scene has significantly varying depths, the blurring is spatially varying such that different regions have different blurring effect. Mathematically speaking, such a blurring process is a spatially varying process which can be expressed as

$$f = Kg + n$$

where *K* is a banded matrix. For spatially invariant case, each row of *K* corresponds to the same low-pass filter up to a spatial shift while each row of *K* may correspond to a different low-pass filter for spatially invariant blurring process. In [6], a two-stage approach is proposed to solve such a spatially varying blind de-blurring problem. The basic idea is to approximate the spatially varying blurring process by a piece-wise spatially invariant blurring process, followed by a PCA-based interpolation process to derive the whole matrix *K*. The key component in the proposed two-stage framework is a non-blind image de-convolution technique [6], which is robust to kernel error, which is unavoidable in spatially varying blind de-blurring. The basic idea of robust image de-convolution method proposed in [7] is to simultaneously estimate three components: (1) sharp image, (2) the distortion of image gradients, and (3) the artifacts caused by kernel error. All three components can be sparsely approximated under different transforms, namely, the sharp image is sparse in wavelet tight frame, the distortion of image gradients is sparse in space, and the artifacts are sparse in local Cosine transform.

References

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