Quantitative Finance and Risk Management

by **Steven Kou**

Introduction

A common misperception, especially before the recent financial crisis, is that quantitative finance and risk management focus on two research areas, option pricing and portfolio optimization. Here I will discuss five of my research projects related to the recent financial crisis: (1) robust risk measures for Basel accords; (2) limit order books; (3) profit sharing in hedge funds; (4) real estate securities; (5) jump risk during the crisis. Note that none of them involves either option pricing or portfolio optimization.

Mean or Median: Robust Risk Measures for Basel Accords

Elementary statistics teaches us that both mean and median measure the average size of a random quantity, but they have different properties. In particular, if we want to obtain a robust measurement, then median is a better choice than mean. Now what does this have to do with trading book capital requirements? In the consultative document released by the BIS (Bank of international Settlement) on 3 May 2012, it is stated that the proposal is to "[move] from value-at-risk (VaR) to expected shortfall (ES), a risk measure that better captures tail risk." We have serious concerns about this proposal. In particular, in Kou, Peng, and Heyde [1] we provide an axiomatic framework to justify VaR.

First of all, if we want to capture the tail risk, e.g., the size of the loss beyond the 99% level, we can either use ES at 99% level, which is tail conditional mean at 99%, or, alternatively, median shortfall (MS) at 99% level, which is defined as the median of the conditional distribution of the loss given that the loss exceeds the 99% VaR. The MS at 99% level is simply equal to VaR at 99.5% level if the underlying loss distribution is continuous, and the two tend to be very close even if the distribution is not continuous. Hence, just like ES, MS (equivalently, VaR at a higher level) also measures the riskiness of a position by taking into account both the size and likelihood of losses.

Secondly, robustness is indispensable for regulatory risk measures. In the internal models-based approach for determining
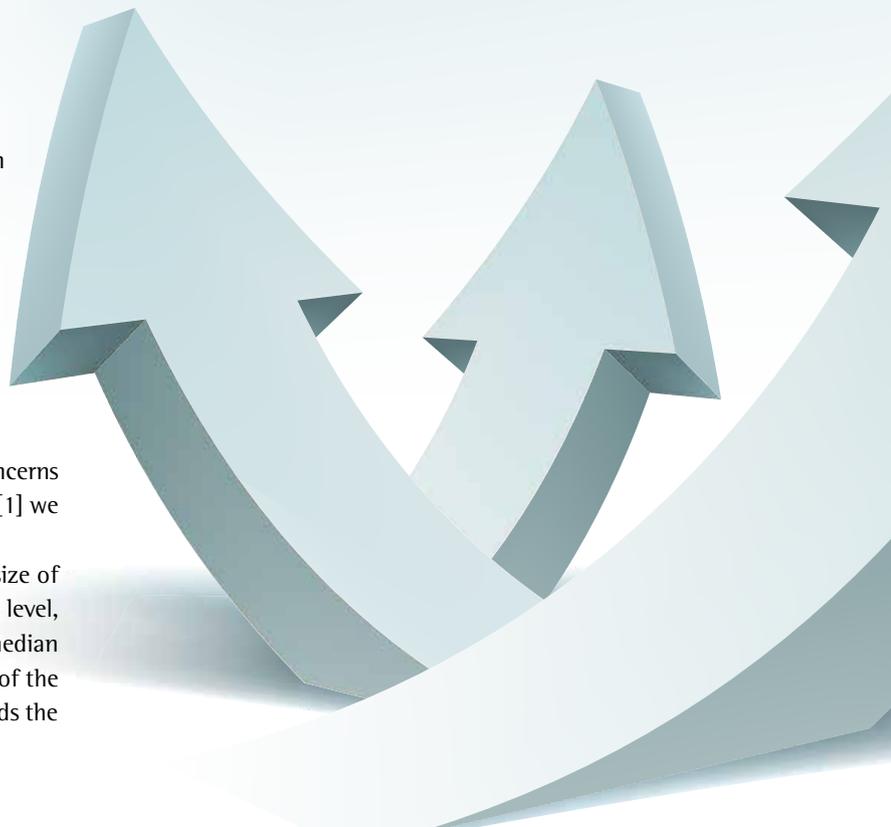
trading book capital requirements, regulators impose the risk measure and allow institutions to use their own internal risk models and private data in the calculation. From a regulator's viewpoint, a regulatory risk measure must be unambiguous, stable, and capable of being implemented consistently across all the relevant institutions, no matter what internal beliefs or internal models each may rely on. The external risk measure should be robust; otherwise, different institutions can be required to hold very different regulatory capital for the same risk exposure, which makes the risk measure unacceptable to both the institutions and the regulators. The requirement of robustness for regulatory risk measures is not anything new; in general, robustness is essential for law enforcement, as is implied by legal realism, one of the basic concepts of law.

Limit Order Books

Compared to dealer markets, traders in limit order markets can submit either limit orders or market orders, leading to more flexibility and transparency. Limit order books is important for understanding high frequency trading, which is of great concern to Singapore Exchange. Of great importance for limit order books is market depth, which is the average number of limit order shares placed per unit price. In fact, market depth is a commonly used indicator for limit order books. In Chen, Kou and Wang [2], we propose a model for limit order books with stochastic, reverse U-shaped, market depth, consistent with empirical studies. Stochastic market depth is necessary to accommodate various order activities, such as limit order submission at and outside the best quotes and order cancelation, which may account for a large proportion of limit order activities. To show the analytical tractability of the model, in addition to a dynamic programming formulation of the optimal execution problem, we attempt to provide easily computable and tight upper and lower bounds for the optimal execution cost, as well as their resulting trading strategies via quadratic programming and jump linear quadratic control.

Profit Sharing in Hedge Funds

Many people believe that a cause of the recent financial crisis is the limited liability of fund managers, which means profits are shared but not losses. This has serious consequence in terms of risk taking. For example, suppose one has a trading strategy that leads to 20% gains 99% of the time. Then it is not clear at all whether a rational individual should use the strategy, as those 1% cases may lead to huge losses. However, it is optimal for fund managers and corporations to pursue such a strategy due to the limited liability protection. Since the limited liability protection is a fundamental principle of firm structure, there is no way to eliminate it.

But one can still have some checks and balances. For example, the fund managers can set up a deposit, such as 10% of fund capital, by using the manager's own money, and if there is a loss, the deposit money will be used first to offset the losses; in return the fund manager can ask for higher profit sharing such as 40%, versus the current hedge fund standard of 20%. The above compensation scheme is called the first-loss scheme. It has been quite popular in China for privately held funds (which are similar to hedge funds in the U.S.), and is emerging in the U.S. according to a CBS Marketwatch report on 23 May 2011.

In He and Kou [3], we analyze both the first loss scheme and the traditional scheme, in which the fund managers invest 10% regular capital and get 20% profit sharing. By using the s-shaped utility function from behavioral finance, we find that if the 10–30 first-loss scheme (i.e. 10% as the first loss capital in return for profit sharing at 30%) is used, then both fund managers and investors will be better off than under the traditional scheme, as measured by their utility functions. Furthermore, the risk of the hedge fund strategy is lower in the 10–30 first-loss scheme.

Real Estate Securities

To a large extent, the financial collapse in 2007–2009 was the result of a national real estate market that proved to be much more tightly interconnected — much more systemically risky — than had previously been recognized. This market is also closely linked to the public sector: the U.S. Federal Reserve, which traditionally held only Treasury securities, currently holds over US$1 trillion in mortgage-backed securities; and the Treasury's actions in the crisis made explicit the government guarantee of debt securities issued by Fannie Mae and Freddie Mac. The creation and continued existence of these government sponsored enterprises reflects the public interest in supporting home ownership; but the near collapse of these giants also reflect large-scale failures of risk management.

Spatial interaction has been well-studied in the spatial econometrics literature. Instead of studying prices of houses and apartments that are illiquid and difficult to be sold short, we study the risk and return of real estate securities that are liquid and can be easily shorted, such as futures contracts on the S&P/Case-Shiller Home Price Indices (CSI Indices). More precisely, in Kou, Peng, and Zhong [4], we attempt to connect spatial econometrics, which emphasizes the statistical modeling of spatial interaction,

with classical asset pricing models including the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT), which characterize risk-return relationship of financial assets that can be freely traded, especially be sold short. Furthermore, we give rigorous econometric analysis of the models by deriving identifiability conditions for the parameters and asymptotic properties of estimators and studying test statistics needed for implementing the models. Finally, an empirical study of the futures contracts on S&P/CSI Indices shows that the spatial APT is not rejected and the spatial interaction parameter is statistically significant.
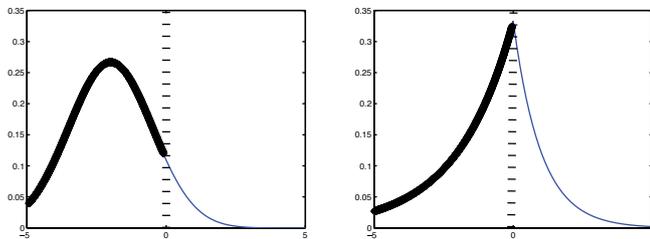

Fig. 1. Non-monotonicity of the normal density (left) and the monotonicity of the double exponential density (right).

Jump Risk during the Crisis

It is well known that jump risk affects equity returns significantly. In Kou, Yu, and Zhong [5], we attempt to answer two questions about jumps in equity indices: (i) How did jumps in equity index returns change during the financial crisis in 2007–2011; in particular, were there significant changes in jump rates or in jump sizes, or both? (ii) Were there finite number of large jumps or infinite number of small jumps in equity returns before and during the crisis?

For the first question, the increases of jump rates when market is in distress, especially for the 1987 crash and the tech-bubble burst around 2001–2002, are well documented. However, whether there are significant changes in jump sizes during financial crises has not been addressed in the existing literature. Also, the previous empirical studies in general do not distinguish positive and negative jump rates.

Based on the latest data on S&P 500 daily returns up to December 2011, we find both positive and negative jump rates increased significantly during the financial crisis, while, somewhat surprisingly, there is little evidence that average jump sizes changed before and after the crisis.

The results in the existing literature regarding the second question are mixed; furthermore, most of the empirical studies on the subject assume normally distributed jumps in equity returns. We found that a simple affine jump-diffusion model with both stochastic volatility and double exponential jump sizes [6] in returns fits S&P 500 daily return data well before and during

the crisis. Therefore, our answer to the second question is that there seems to be finite number of large jumps in equity returns. In short, affine jump-diffusion model with a proper jump size distribution can fit equity return data well both before and after the crisis. Intuition behind this lies in the differences between the two jump-size distributions.

In terms of small jumps, the normal distribution does not have monotone structure. For example, it can be seen from Figure 1 (normal density on the left and double exponential density on the right) that, if jump sizes are normally distributed with mean -2%, jumps down -2% is more likely to occur (i.e. has a higher density) than jumps down just -1%. The monotone structure of double-exponential jumps provides better model fit of small jumps in equity returns.

In terms of large jumps, the double-exponential distribution is also suitable because it has heavy tails. Heyde and Kou [7] show that it is difficult to distinguish exponential-type tails from power-type tails from empirical data. For example, far at the right-tail, say, at the probability of 0.01%, the population quantile of the exponential distribution is 6.02, which is larger than 5.97, the population quantile of normalized Student-distribution with degree of freedom 7, even though we know that the Student-t distribution should have an asymptotically heavier tail than the exponential-type distribution. The heavy-tail feature of the double exponential distribution helps to fit the large jumps during the crisis. 🛈

References

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