Research Highlight: Moment map flows and the Hecke correspondence for quivers

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Morse theory relates topological information about a manifold (for example, homology and cohomology groups) to analytical information (critical points of functions defined on the manifold). Modern versions of Morse theory incorporate information about flow lines between critical points to obtain more topological information (for example, the cup product in the cohomology ring). A particularly interesting example is the space of representations of a quiver with relations, for which the topology contains information about representations of algebras used in physics, such as quantum affine algebras and W-algebras [N1], [N2], [N3]. One of the keys to this construction is a variety called the Hecke correspondence, which interpolates between to different moduli spaces of stable representations of a quiver. In recent work, Dr. Graeme Wilkin [W] has proved a new Morse-theoretic interpretation of the Hecke correspondence in terms of flow lines between critical points of a moment map function.

References:

[N1] Hiraku Nakajima. Quiver varieties and finite-dimensional representations of quantum affine algebras. {\em J. Amer. Math. Soc.}, 14(1):145--238 (electronic), 2001.

[N2] Hiraku Nakajima. Quiver varieties and {t}-analogs of {q}-characters of quantum affine algebras. {\em Ann. of Math. (2)}, 160(3):1057--1097, 2004.

[N3] Hiraku Nakajima. Handsaw quiver varieties and finite {W}-algebras. {\em Mosc. Math. J.}, 12(3):633--666, 669--670, 2012.

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