Research Highlight: Jordan property for automorphism groups of projective varieties

Work of Professor ZHANG De-Qi

A century ago, Camille Jordan proved that the complex general linear group $GL_n(C)$, has the Jordan property: there is a constant C_n such that every finite subgroup of $GL_n(C)$ has an abelian subgroup of index at most C_n.

In [1], Popov asks whether the full automorphism group Aut(X) of a projective variety has the Jordan property. When X is 2-dimensional, it is true; see [1] and [2].

In [4] Prof. Zhang and his PhD student Meng first show that all algebraic groups have the Jordan property. They then answer Popov's question in the affirmative in the full generality.

The proof uses Minkowski's bound for finite linear groups over a field finitely generated over the rationals, and the classical decomposition of Rosenlicht of an algebraic group into the product of its linear or anti-linear part.

See also [3] for the case of biraitonal automorphisms, extending Serre's result in dimension two.

References

[1] V. L.Popov, Jordan groups and automorphism groups of algebraic varieties, in: Automorphisms in birational and affine geometry, 185-213, Springer Proc. Math. Stat., 79, Springer, Cham, 2014.

[2] Y. G. Zarhin, Jordan groups and elliptic ruled surfaces, Transform. Groups, 20 (2015), no. 2, 557-572.

[3] Y. Prokhorov and C. Shramov, Jordan property for groups of birational selfmaps, Compos. Math. 150 (2014), no. 12, 2054-2072.

[4] Sheng Meng and De-Qi Zhang, Jordan property for non-linear algebraic groups and projective varieties, American Journal of Mathematics, Volume 140, Number 4, August 2018, pp. 1133-1145.