Research Highlight:
Global boundedness of solutions to a chemotaxis-haptotaxis model

Work of Associate Professor Peter Pang

In an invited article [1], Pang and his co-author Wang Yifu (Beijing Institute of Technology) continued their investigation into cancer invasion models that involve the biochemical mechanisms chemotaxis and haptotaxis [2].

In this investigation, they focused on integrative interactions of chemotaxis, haptotaxis, logistic growth of cancer cells and remodelling of the extra-cellular matrix. The mathematical model comprised two parabolic partial differential equations and an ordinary differential equation in a strongly coupled system. The authors noted that in previous studies, remodelling was often neglected, citing its suspected low rate.

In spatial dimension 2, the authors improved earlier known results. In particular, they removed the assumption that the matrix degrading enzyme diffused at a much slower rate than the cancer cells [3], and the restriction to the high cell proliferation regime [4].

In dimension 3, the mathematical problem was rather delicate. Previous known results indicated that solutions might have weak regularity properties [5, 6, 7]. The authors successfully established that the solutions in fact enjoyed a high degree of regularity, and showed furthermore that the solutions were well behaved, in that they were globally bounded in the low cell proliferation regime.

References

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