Research Highlight: Low-dimensional lonely branching random walks die out

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The lonely branching random walk is a model of interacting particles on the d-dimensional integer lattice Z^d, where each particle moves on the lattice Z^d as a random walk independent of all other particles. When a particle is alone at its position (hence the name lonely branching random walk), the particle undergoes critical binary branching, namely with equal probability, it either dies or branches into two particles. Starting with a spatially homogeneous particle configuration, say with 1 particle at each lattice point, the question is whether in any finite region, the number of particles in that region will converge in probability to 0, in which case the model is said to die out; otherwise the model is said to survive.

Previously, it was known that in spatial dimensions 3 and higher, the lonely branching random walk model survives. In this work, R. Sun and his coauthor M. Birkner proved the conjecture that in spatial dimensions 1 and 2, the lonely branching random walk model dies out. This dichotomy between extinction in dimensions 1 and 2 and survival in dimensions 3 and higher was previously known for independent critical branching random walks. The result of Birkner and Sun shows that the dichotomy is robust and applies also to a large class of dependent critical branching random walks.

References:

M. Birkner, **R. Sun**, “Low-dimensional lonely branching random walks die out”. Annals of Probability, 47, No.2 (2019): 774-803.