Research Highlight: The inductive strength of Ramsey's theorem for pairs

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Ramsey's theorem states that that for natural numbers n, k, RT^n_k holds, where RT^n_k states: if the n-element subsets of natural numbers are colored in one of k colors, then there is an infinite set all of whose n-element subsets have the same color.

Logical analysis of Ramsey's theorem is based on a system called RCA_0. It is known that when $n > 2$, and $k > 1$, RT^n_k implies the arithmetic comprehension axiom (Jockusch 1972). Since the case $k = 1$ is trivial, the proof-theoretic strength, in particular the inductive strength, of RT^2_2 became a major open problem when it was officially posed in 2001 (although the problem was known some years before that). The main result of this paper says that RT^2_2 does not prove $\sum_2$-induction, thus provided a tighter upper bound of its first-order strength.

Reference:

C.T. Chong, T. Slaman and Y. Yang, “The inductive strength of Ramsey's Theorem for Pairs”. Advances in Mathematics, 308 (2017): 121-141.