

## Research Highlight: The order structure of special ultrafilters on the natural numbers

### Work of

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An ultrafilter can be thought of as a finitely additive two-valued measure on the subsets of the natural numbers. A general theory of convergence in topological spaces can be developed using ultrafilters. For this, it is necessary to be able to compare different ultrafilters in terms of their fineness in detecting limiting behaviors in topological spaces. The Rudin-Keisler and Tukey orderings on ultrafilters were developed as a means for such comparison. Additionally, special classes of ultrafilters that are particularly fine at detecting convergence were identified.

In 1973, Blass [2] asked what is the order structure of the class of  $P$ -points under the Rudin-Keisler ordering. In [1], we show under suitable additional axioms that this class has a very complicated structure. In particular, it is possible to embed every partial order of size less than continuum into this class. By another result of Shelah, additional axioms are necessary for such results.

### References

- [1] Dilip Raghavan and Saharon Shelah. *On embedding certain partial orders into the  $P$ -points under Rudin-Keisler and Tukey reducibility*, Trans. Amer. Math. Soc. **369** (2017), 4433 – 4455.
- [2] Andreas Blass. *The Rudin-Keisler ordering of  $P$ -points*, Trans. Amer. Math. Soc. **179** (1973), 145 – 166.