Research Highlight: Unipotent Representations of Real Classical Groups

Work of Professor ZHU Chengbo

A fundamental problem in representation theory is to determine the unitary dual of a given Lie group G, namely the set of equivalent classes of irreducible unitary representations of G. A principal idea, due to Kirillov and Kostant, is that there is a close connection between irreducible unitary representations of G and certain geometric objects called coadjoint orbits (which are the orbits of G on the dual of its Lie algebra). This is known as the orbit method. Due to its resemblance with the process of attaching a quantum mechanical system to a classical mechanical system, the process of attaching a unitary representation to a coadjoint orbit is also referred to as quantization in the representation theory literature.

The orbit method has achieved overwhelming success in the context of nilpotent and solvable Lie groups. For more general Lie groups, work of Mackey and Duflo suggest that one should focus attention on reductive Lie groups. As expounded by Vogan in his writings, the problem finally is to quantize nilpotent coadjoint orbits in reductive Lie groups. The ``corresponding'' unitary representations are called unipotent representations.

In a recent work joint with Jia-Jun Ma and Binyong Sun, Professor Zhu has constructed all unipotent representations associated to certain classes of nilpotent coadjoint orbits of real classical groups (in the sense of Barbasch-Vogan), by profitably combining analytic ideas of Howe on theta lifting and algebro-geometric ideas of Vogan on associate varieties.

Reference: J.-J. Ma, B. Sun and C.-B. Zhu, “Unipotent representations of real classical groups”, arXiv:1712.05552.