Research Highlight: The Theta Correspondence

Work of Professor GAN Wee Teck

Theta correspondence is a technique for transferring representations or automorphic forms from one group to another. It has been a useful tool in the Langlands program as it gives an efficient way of constructing automorphic forms with desired properties. The subject was initiated by Roger Howe in the early 1970's, building on earlier work of Hermann Weyl, André Weil and others.

For it to be effective as a tool for constructing automorphic forms, several fundamental problems in the theory needs to be resolved. One of these is the so-called Howe duality conjecture. Howe himself proved some cases and Waldspurger settled almost all cases in 1990, but the full conjecture remains unresolved till recently. In the past two years, Prof. Gan and his collaborators have settled several of these fundamental problems [4,3,1] and pushed the boundary of applications of theta correspondence. In particular, in [2], Gan and Ichino used the theta correspondence to resolve the Fourier-Jacobi case of the local Gross-Prasad conjecture for unitary groups.

References

[1]   H. Atobe and W.T. Gan, Local theta correspondence of tempered representations and Langlands parameters, to appear in Inventiones Math (2017).
[2]   W.T. Gan and A. Ichino, The Gross-Prasad conjecture and local theta correspondence, Inventiones Math. 206 (2016), no. 3, 705-799.
[3]   W.T. Gan and B.Y. Sun, The Howe duality conjecture: quaternionic case, to appear in Contemporary Math 2017 (a volume in honor of R. Howe's 70th birthday).
[4]   A proof of the Howe duality conjecture (with S. Takeda), J. of American Math. Society 29 (2016), no. 2, 473-493.