Research highlight: Nodal-continuous finite element methods of very weak solutions of curlcurl-graddiv eigenvalue interface problems

Work of Associate Professor TAN Choon Ee, Roger

Curlcurl-graddiv eigenvalue interface problems arise from many important physical problems, such as Maxwell equations, Navier-Stokes equations, fluid-structure interactions, magnetohydrodynamics, Ginzburg--Landau equations, etc. Such curlcurl-graddiv interface problems generally have very weak and singular solutions of very low piecewise regularity (e.g., non-square-integrable gradients). Widely known, the very weak solutions bring about many difficulties in the design and development of finite element methods. There are two main challenges. Although the curlcurl-graddiv problem is a second-order elliptic problem, very closely related to Poisson equations of Laplacian, the classical nodal-continuous Lagrange finite element method fails, resulting in wrong convergent approximations of very weak solutions. When considering eigenvalue problems of wave propagations, the very weak solution further leads to spurious and non-physical eigenmodes, seriously polluting the whole spectrum. Prof TAN Choon Ee, Roger and co-authors proposed and analyzed a new nodal-continuous finite element method, which can highly efficiently solve very weak solutions of curlcurl-graddiv eigenvalue interface problems numerically, with correctly convergent approximations and spurious-free eigenmodes.

Reference: H. Y. Duan, P, Lin, and Roger C E Tan, “A finite element method for a curlcurl-graddiv eigenvalue interface problem”. SIAM Journal on Numerical Analysis, 54(2016), 1193-1228.