Research highlight: The Anti-Field-Descent Method

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Field descent method is a powerful tool to study several combinatorial problems that include difference sets and Barker's sequences. The main idea is to find conditions that certain cyclotomic integers in $\mathbb{Z}[\zeta_n]$ are actually contained in relatively $\mathbb{Z}[\zeta_r]$ for a proper divisor r of n. Very often, we obtain cyclotomic integers with small complex modulus. The main idea is first developed by Schmidt [3] and then furhter extended in [1] small fields.

Recently, Leung and Schmidt [2] develop a method which reveals a complementary phenomenon: certain cyclotomic integers cannot be contained in relatively small fields and thus must have relatively large complex modulus.

This method, in particular, yields progress towards the circulant Hadamard matrix conjecture. The application of the new conditions to previously open cases of Barker sequences shows that there is no Barker sequence of length ℓ with $13 < \ell \leq 4 \cdot 10^{33}$. Furthermore, 229,305 of the 237,807 known open cases of the Barker sequence conjecture are ruled out.

References

[1] K. H. Leung, B. Schmidt: The Field Descent Method. *Des. Codes Crypt.* **36** (2005), 171–188.

[2] K. H. Leung, B. Schmidt: The Anti-Field-Descent Method. *Journal of Combinatorial Theory Ser. A* **139** (2016), 87-131.

[3] B. Schmidt: Cyclotomic integers and finite geometry. *Journal of American Mathematical Society* **12** (1999), 929–952.