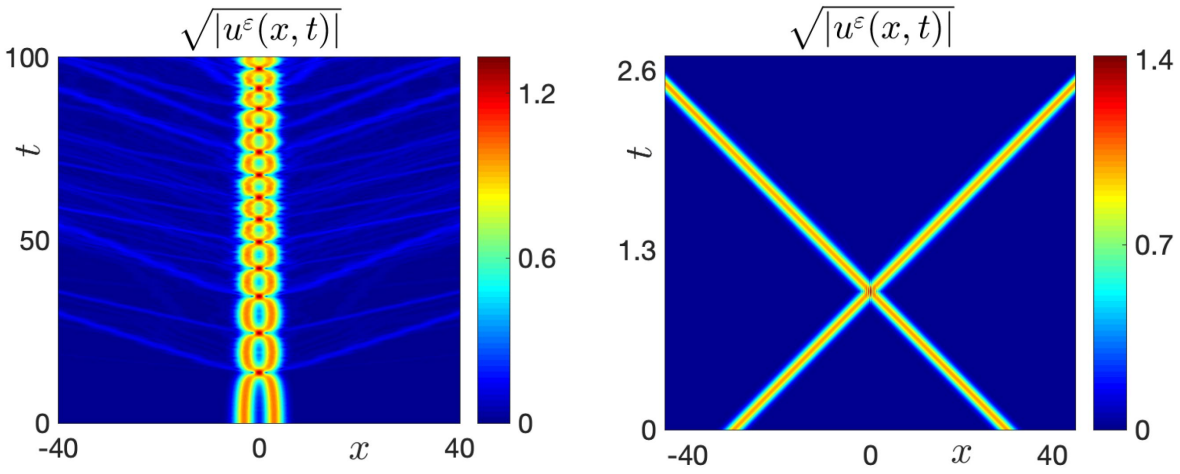
Research Highlight:
Numerical Methods for the Logarithmic Schrodinger Equation

Work of Professor BAO Weizhu

Prof BAO Weizhu and co-authors proposed different regularized numerical methods and establish their optimal error bounds for the logarithmic Schrodinger equation (LogSE). In fact, the LogSE arises from different applications, such as quantum mechanics, quantum optics, transport and diffusion phenomena, open quantum systems, nuclear physics, Bohmian mechanics, effective quantum gravity, theory of superfluidity, and Bose-Einstein condensation.

Due to the blow-up of the logarithmic nonlinearity, there are significant difficulties in designing numerical methods and establishing their error bounds for the LogSE. In order to suppress the round-off error and to avoid blow-up, a regularized logarithmic Schrodinger equation (RLogSE) was proposed with a small regularization parameter and linear convergence was established between the solutions of RLogSE and LogSE in term of the small regularization parameter [1]. A semi-implicit finite difference method was presented for discretizing the RLogSE and error estimates were established in terms of the mesh size and time step as well as the small regularization parameter [1]. Then we used the Lie-Trotter splitting integrator to solve the RLogSE and established an improved error bound for the LogSE by the Lie-Trotter splitting method [2]. In addition, the CNFD was also applied to discretize the RLogSE, which conserves the mass and energy in the discretized level. Numerical results were reported to confirm our error bounds and to demonstrate that they are sharp [2]. Finally, our numerical methods were applied to study numerically the dynamics of the LogSE including interaction of static and moving Gaussons. In fact, the regularization technique and different numerical methods can be applied to solve different partial differential equations with logarithmic nonlinearity.



Interactions of two static (left) and moving (right) Gaussons under the dynamics of the LogSE

Reference:

[1] W. Bao, R. Carles, C. Su and Q. Tang, “Error estimates of a regularized finite difference method for the logarithmic Schrodinger equation”. SIAM Journal on Numerical Analysis, 57 (2019): 657-680.

[2] W. Bao, R. Carles, C. Su and Q. Tang, “Regularized numerical methods for the logarithmic Schrodinger equation”, Numerische Mathematik, 143 (2019): 461-487.