Research Highlight: Gaussian Complex Zeros, Equilibrium Measures and Forbidden Regions

Work of Assistant Professor Subhro GHOSH

Consider the Gaussian entire function

$$F_{\mathbb{C}}(z) = \sum_{k=0}^{\infty} \xi_k \frac{z^k}{\sqrt{k!}},$$

where $\{\xi_k\}_{k=0}^{\infty}$ is a sequence of independent standard complex Gaussians. This random Taylor series is distinguished by the invariance of its zero set with respect to the isometries of the plane $\mathbb{C}$. It has been of considerable interest to study the statistical properties of the zero set, particularly in comparison to other planar point processes.

We show that the law of the zero set, conditioned on the function $F_{\mathbb{C}}$ having no zeros in a disk of radius r and normalized appropriately, converges to an explicit limiting Radon measure on $\mathbb{C}$ as $r \to \infty$. A remarkable feature of this limiting measure is the existence of a large *forbidden region* between a singular part supported on the boundary of the (scaled) hole and the equilibrium measure far from the hole. In particular, this answers a question posed by Nazarov and Sodin, and is in stark contrast to the corresponding result of Jancovici, Lebowitz, and Manificat in the random matrix setting: there is no such forbidden region for the Ginibre ensemble.

References

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