

Research Highlight: A surface with discrete and non-finitely generated automorphism group

Work of Professor DINH Tien Cuong and Professor Keiji OGUISO (Tokyo University)

Let X be a real algebraic variety defined by real polynomials. By considering the complex roots of these polynomials, we obtain a complex algebraic variety which is called a complexification of X .

A given complex algebraic variety may have several non-equivalent real forms. All complex algebraic varieties of dimension 1 and large classes of varieties in higher dimensions only admit finitely many non-equivalent real forms.

In collaboration with Prof. Keiji OGUISO (Tokyo University), Professor DINH Tien Cuong (National University of Singapore) proved that there are varieties of arbitrary dimension at least 2 having infinitely many non-equivalent real forms. Moreover, their groups of automorphisms are countable and non-finitely generated. This result solves a longstanding question which has been considered by Borel-Serre, Degtyarev-Itenberg-Kharlamov and Benzerga. The case of dimension at least 6 was obtained by Lesieutre.

Reference:

- [1] Dinh T.-C., Oguiso K., A surface with discrete and non-finitely generated automorphism group, *Duke's math. J.* 168 (2019), no. 6, 941-966.
- [2] Dinh T.-C., Oguiso K., Yu X., Projective rational manifolds with non-finitely generated discrete automorphism group and infinitely many real forms, preprint (2020). arXiv:2002.04737