Research Highlight: Automorphisms of two-generator free groups and spaces of isometric actions on the hyperbolic plane

Work of Professor TAN Ser Peow

The automorphisms of a two-generator free group G acting on the space of orientation-preserving isometric actions of G on hyperbolic 3-space defines a dynamical system. Those actions which preserve a hyperbolic plane but not an orientation on that plane is an invariant subsystem, which reduces to an action of a group Γ on $\mathbb{R}^3$ by polynomial automorphisms preserving the cubic polynomial

$$P(x, y, z) := -x^2 - y^2 + z^2 + x\,y\,z - 2$$

and an area form on the level surfaces of $P(x, y, z)$.

In [1], the connection between geometric structures on various hyperbolic surfaces of small complexity, and the dynamics of the action of Γ was explored, which shows the close connection between spaces of geometric structures and dynamics of the action of certain related automorphism groups.

Reference:

[1] Goldman W, McShane G, Stantchev G, Tan SP, Automorphisms of two-generator free groups and spaces of isometric actions on the hyperbolic plane. Memoirs of the American Mathematical Society 259 (1249):1-92