Research Highlight: Are Complete Information Dynamic Games Solvable?

Work of Professor SUN Yeneng

A complete information dynamic game is a game played in multiple time periods with the payoffs being known to all the players. If, in addition, every move is observed by all the players, the game is said to have perfect information. The first major result on alternating-move games with perfect information was Zermelo's Theorem (1913): In any two-person zero-sum game in which the players observe the history and payoffs, and move alternatingly with finitely many choices (like Chess and Go) in finitely many stages, either one of the players has a winning strategy, or both players can individually ensure a draw; namely, the game is solvable. Zermelo's method still works for general *deterministic* dynamic games with perfect information. However, when uncertainty is brought into such games, it may not be solvable in general.

In their 2020 paper, Prof SUN Yeneng and his co-author HE Wei (2014 NUS Ph.D in Mathematics) show that general dynamic games with perfect information are solvable under the assumption of atomless state transitions (whenever uncertainty appears).

For simultaneous-move dynamic games with complete information, Selten's 1965 Nobel Prize winning work shows that such games are solvable (in terms of subgame perfect equilibrium) in the case of *finite actions*, which is a standard result in undergraduate game theory textbooks. However, such a result fails in general when the players are allowed to have continuous choice sets. It is shown in the same paper of He and Sun (2020) that simultaneous-move dynamic games with complete information and continuous action spaces are solvable if every time period has atomless state transition.

References:

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Ernst Zermelo, On an application of set theory to the theory of the game of Chess, *Proceedings of the Fifth International Congress of Mathematicians* II, Cambridge University Press, 1913, 501-504.