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Research Highlight: Burnett spectral method for the Boltzmann equation

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The Boltzmann equation is one of the fundamental models in the gas kinetic theory, which describes the state of the gas by considering the distribution of gas molecules in the position-velocity space. Since both the position and the velocity are three-dimensional variables, the Boltzmann equation turns out to be a six-dimensional problem, which is highly costly to solve numerically. Moreover, since the velocities of molecules change due to collisions, there is a collision term in the Boltzmann equation, containing a quadratic operator with a five-dimensional integral. This also introduces significant difficulties in the numerical simulation.

To overcome the difficulties, we choose to discretize the velocity space using the spectral method. The basis functions we adopt are based on the representation of orthogonal polynomials under the spherical coordinate system, known as the Burnett polynomials. By utilizing the rotational invariance of the collision operator, the representation of the collision operator under such basis has some sparsity, allowing us to reduce the computational cost. More importantly, the spectral coefficients correspond to the moments of the distribution function. Since we are mainly concerned about lower-order moments such as density and temperature, the higher-order moments can be processed based on simpler collision models, leading to considerable savings of the computational cost.

Reference:

Z. Hu and Z. Cai, Burnett Spectral Method for High-Speed Rarefied Gas Flows, SIAM Journal on Scientific Computing, 42(5): B1193–B1226, 2020.