



Research Highlight: Small Scale Formations in the Incompressible Porous Media Equation

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The incompressible porous media (IPM) equation describes the evolution of density transported by an incompressible velocity field given by Darcy's law. Here the velocity field is related to the density via a singular integral operator, which is analogous to the 2D surface quasi-geostrophic (SQG) equation. The question of global regularity vs finite-time blow-up remains open for smooth initial data, although numerical evidence suggests that small-scale formation can happen as time goes to infinity.

In this paper, we construct examples of solutions to the IPM equation that must exhibit infinite in-time growth of derivatives provided they remain smooth. As an application, this allows us to obtain nonlinear instability for a class of stratified steady states of IPM, even if the steady state is linearly stable.

References:

Kiselev, A., Yao, Y. Small Scale Formations in the Incompressible Porous Media Equation. *Arch Rational Mech Anal* **247**, 1 (2023). <https://doi.org/10.1007/s00205-022-01830-z>