MA4198 PROJECT PROPOSAL (PROJECT CUM SEMINAR GROUP)

SUPERVISOR'S INFO

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PROJECT ID: PS2320-10

TITLE

Lie Algebras G2

BRIEF DESCRIPTION OF PROJECT

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In this project, we will learn some basics of Lie algebras, with a special emphasis on exceptional Lie algebra, particularly of type g2. Our journey will start with an exploration of finite-dimensional Lie algebras, leading us to the classification of simple Lie algebras. Following this groundwork, we'll transition into constructing and understanding the nuanced structure of Lie algebras of type g2.

Outlined below are the proposed (individual) topics for participating students:
1. Delving into Octonion and Jordan algebras.
2. Unveiling the basics of Lie Algebra g2, encompassing its root systems, Weyl groups, and Chevalley basis.
3. Identifying and classifying the Nilpotent orbits of Lie Algebra g2.
4. Understanding the centralizers of nilpotent elements along with their component groups.
5. Examining the maximal subalgebra of g2.
6. Tackling advanced explorations including the classification of nilpotent orbits of g2 over finite fields, real fields, and p-adic fields, and embarking on the construction of F4 along with its associated inquiries as above.

A distinctive feature of this project is the integration of ChatGPT alongside Mathematica/Sagemath to aid us in executing matrix computations and delving deeper into the proposed topics. While familiarity with Mathematica or Sagemath is not a prerequisite, ChatGPT will serve as a valuable companion in not only facilitating code formulation within Mathematica/Sagemath but also in enhancing comprehension of the mathematical content we'll be exploring.

EXPECTATION/S

The objectives of this project are threefold:
1. **Enhancing Independent and Collaborative Learning Skills:**
This project is meticulously structured to encourage students to embark on self-driven mathematical explorations while also fostering a collaborative learning environment with peers. With a rich repository of detailed references, students are encouraged to grasp the basics independently. The supervisor's role is envisioned more as a supportive guide rather than a traditional lecturer, thus promoting a culture of self-reliance and peer-to-peer learning.

2. **Mastering Fundamental Concepts of Finite Dimensional Lie Algebras:**

A core aim is to equip students with a solid understanding of the basic concepts surrounding simple finite dimensional Lie algebras and their classifications. This foundation is crucial for the deeper explorations and applications that follow.

> **3. Acquiring Proficiency in Utilizing Mathematica or Sagemath (with ChatGPT):**

Gaining hands-on experience with Mathematica or Sagemath is integral to this project as these tools will be employed to verify certain structural properties of Lie algebras. Students will learn how to harness the computational power of these platforms to bolster their theoretical understanding and practical investigations.

PREREQUISITE/S (at level 3000 or below, with at most one course at level 3000)

MA3201 Algebra II (includes MA201 and MA2202 or their S-versions)
MA2214 Combinatorics and Graphs I

READING REFERENCE/S

Introduction to Lie Algebras and Representation Theory by James E. Humphreys
Octonions, Jordan Algebras, and Exceptional Groups by Springer and Veldkamp
Lectures on Chevalley Groups by Steinberg
Mathematica: https://www.wolfram.com/mathematica/
Sagemath: https://www.sagemath.org/
ChatGPT: https://chat.openai.com/chat