

MA4198 PROJECT PROPOSAL (PROJECT CUM SEMINAR GROUP)

SUPERVISOR'S INFO

Name:	Huanchen Bao
Email:	huanchen@nus.edu.sg
Tel number:	66012430
Office location:	S17-06-17

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TITLE

Towards the formalization of Coxeter combinatorics

BRIEF DESCRIPTION OF PROJECT

Proof assistants are specialized software tools that verify the correctness of proofs by ensuring they adhere strictly to logical foundations. Among these, Lean 4, chiefly developed by Leonardo de Moura, has garnered significant attention in the mathematical community for its robust architecture and capabilities. Unlike computational software such as MATLAB, Lean 4 is a proof assistant software that checks the proofs are correct down to their logical foundation. For example, while MATLAB can compute the derivative of a particular function, Lean 4 can check the correctness of a given proof of continuity. The language has attracted a lot attentions from the mathematical society in recent years; for example, Kevin Buzzard initiated a massive project on April 2024 to formalize Fermat's Last Theorem using Lean 4.

Coxeter groups are groups generated by reflections; for example, the symmetry group of each regular polyhedron is a finite Coxeter group. They are generalizations of symmetric groups and Dihedral groups. Coxeter groups find applications in many areas of mathematics.

This project aims to formalize combinatorial results on Coxeter groups using Lean 4. This project shall focus on the programming aspect. We first focus on the textbook "Mathematics in Lean" learning the proof assistant Lean 4. Then we shall introduce some basic results Coxeter groups. Finally, we aim to formalize the results in Lean 4. There will be another parallel project on the combinatorics of Coxeter groups by Prof. Zhang Lei focusing on the mathematical aspect.

EXPECTATION/S

This project has a significant component on programming using Lean 4. Students are expected to formalize some results related with combinatorics of Coxeter groups. They could be small lemmas, or some non-trivial theorem as a combined team work.

PREREQUISITE/S (at level 3000 or below, with at most one course at level 3000)

MA 2202 Algebra I, MA 3201 Algebra II

READING REFERENCE/S

Mathematics in Lean by Jeremy Avigad and Patrick Massot

Combinatorics of Coxeter groups by Anders Bjorner and Francesco Brenti