Research Highlight: The Critical 2d Stochastic Heat Flow

Caravenna, Francesco; Sun, Rongfeng; Zygouras, Nikos

(authors listed in alphabetical order)

The solution theory for singular stochastic partial differential equations (SPDE) has witnessed tremendous progress in recent years thanks to the breakthroughs of M. Hairer, Gubinelli-Imkeller-Perkowski et al. But these theories break down at the critical dimension. One classic example is the Stochastic Heat Equation (SHE) at its critical dimension 2, which furthermore exhibits a phase transition. In recent works, the authors constructed a stochastic process called the critical 2d stochastic heat flow (SHF), which gives a meaning to the long sought solution for the SHE in the critical dimension 2 and in the critical window. Furthermore, they showed that the critical 2d SHF is neither Gaussian, nor can it arise as the exponential of a Gaussian field.

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