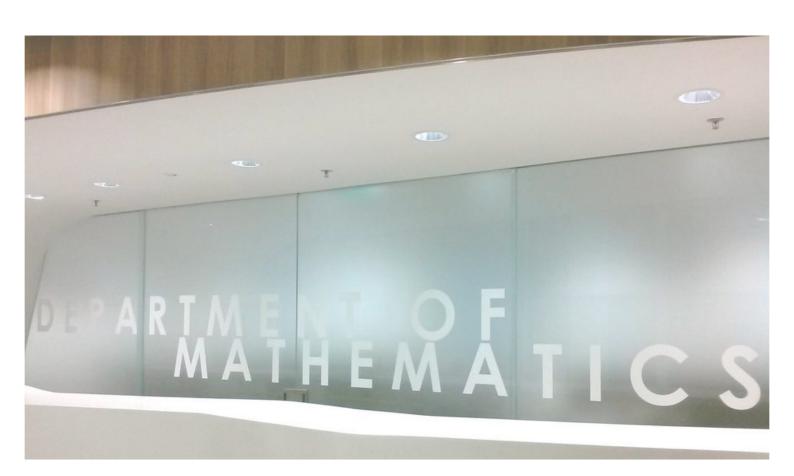# Workshop on Theta Correspondence
(09–12 September 2024)

ORGANIZING COMMITTEE

Edmund Karasiewicz
National University of Singapore

Venue

Department of Mathematics
Block S17, Lvl 04, Room 06
10 Lower Kent Ridge Road
Singapore 119076

For more information: https://24theta.notion.site/24-Hours-of-Theta-60847f460e3b4780a03e85fb29d1ce51

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Programme Schedule

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	1330–1430	Automorphic functional on the space of the minimal representation Nadya Gurevich *Ben-Gurion University of the Negev, Israel*
	1430–1500	*Tea Break*
	1500–1600	An Extension of the conjecture of Prasad to quaternionic dual pairs Hirotaka Kakuhama *Hokkaido University, Japan*

	Time	Title / Speaker
Tuesday 10 September 2024	0930–1030	Theta correspondence over finite fields and its geometrization Jia-Jun Ma *Xiamen University Malaysia, Malaysia*
	1030–1100	*Tea Break*
	1100–1200	Finite theta correspondence, unipotent support and orbit correspondence Shu-Yen Pan *National Tsing Hua University, Hsinchu*
	1200–1430	*Lunch Break*
	1430–1530	Local new forms for unramified even unitary groups Hiraku Atobe *Kyoto University, Japan*
	1530–1600	*Tea Break*
	1600–1700	The local theta correspondence: an infomercial talk Shuichiro Takeda *Osaka University, Japan*

	Time	Title Speaker
Wednesday 11 September 2024	1000–1130	Discussion and collaboration @ Lounge
	1130–1330	*Lunch Break*
	1330–1430	*Location: S17, Lvl 4, Room 04* Theta correspondence and simple factors of global Arthur parameters Chenyan Wu *The University of Melbourne, Australia*
	1430–1500	*Tea Break*
	15:00–15:30	Some theorems in representation theory of classical Lie groups Binyong Sun *Zhejiang University, China*

	Time	Title Speaker
Thursday 12 September 2024	1000–1130	Discussion and collaboration @ Lounge
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	1330–1430	Homological theta lifting and applications Rui Chen *Zhejiang University, China*
	1430–1500	*Tea Break*
	1500–1600	Degenerate principal series representations and real double flag varieties of finite type Kyo Nishiyama *Aoyama Gakuin University Japan*

Friday	
13 September 2024	
1000–1600	Discussion and collaboration @ Lounge

This schedule is accurate as of 09 September 2024 and subjected to changes.

Abstract of Talks

Hiraku Atobe
Kyoto University, Japan

Local new forms for unramified even unitary groups

In 1973, Casselman established the theory of local newforms of GL_2, which bridges a gap between elliptic modular forms and automorphic representations of GL_2 over the rational fields. In this talk, we introduce compact open subgroups of quasi-split even unitary groups over p-adic field for each even non-negative integers, and explain the theory of local newforms for irreducible tempered generic representations with a certain condition on the central characters. To do this, we introduce the Fourier-Jacobi modules of irreducible representations.

Rui Chen
Zhejiang University, China

Homological theta lifting and applications

Following the idea of Adams, Prasad and Savin, one can consider the Ext-analogues of theta lifting. In this talk I will discuss some recent works on this direction, give some applications and exhibit some interesting phenomenon related to local L-functions.

Nadya Gurevich
Ben-Gurion University of the Negev, Israel

Automorphic functional on the space of the minimal representation

The minimal representation Π_v on a p-adic group $G(F_v)$ often affords a model on the space of functions on a p-adic variety X. A maximal parabolic subgroup, whose unipotent radical is either abelian or Heisenberg, acts on functions in this model by explicit geometric formulas. Such model is important in establishing local theta correspondence for dual pairs in G. Local models give rise to a global model of the representation Π of the group of adelic points $G(A)$. The representation Π can be realized automorphically, as residue of some degenerate Eisenstein series. The goal of the talk is to write an automorphic functional on the global model of Π for split groups of type D_n and E_n, stressing the main term and the boundary terms, and their recursive nature.

Hirotaka Kakuhama
Hokkaido University, Japan

An extension of the conjecture of Prasad to quaternionic dual pairs

In this talk, I will formulate a conjecture describing the behavior of the internal structures of L-packets under the local theta correspondence for quaternionic dual pairs, which extends the conjecture of Prasad. To do this, we use the local Langlands conjecture for rigid inner twists. Moreover, I will explain the proof in the Archimedean case. In this case, a description of theta correspondence has already been given in terms of certain parameters that are extensions of Harish-Chandra parameters. Thus, the basic strategy of the proof is to translate them in terms of Langlands parameters.

Jia-Jun Ma
Xiamen University Malaysia, Malaysia

Theta correspondence over finite fields and its geometrization

Let G_1 and G_2 be a reductive dual par. The Harish-Chandra series of G_1 and G_2 correspond to each other under the theta correspondence. With Congling Qiu and Jialiang Zou, we explicitly computed this correspondence by analyzing the relevant Hecke algebra bimodules and applying a Tits deformation argument. This provides another proof of Aubert Michel-Rouquier's conjecture, which was first proved by Shu-Yen Pan. In this talk, we will discuss a geometrization of the picture. Consequently, we obtained a new description of the theta correspondence in terms of the Springer theory.

This is an ongoing joint work with Congling Qiu, Jialiang Zou, and Zhiwei Yun.

Kyo Nishiyama
Aoyama Gakuin University Japan

Degenerate principal series representations and real double flag varieties of finite type

Let $G = \mathrm{Sp}_{2n} \mathbb{R}$ (or $U(n, n)$) and P a maximal parabolic subgroup stabilizing a Lagrangian subspace (Siegel parabolic subgroup). Let $H \simeq \mathrm{GL}_n \mathbb{R}$ be a Levi subgroup of P, which is also a symmetric subgroup of G. We choose a parabolic subgroup $Q \subset H$. Let $X = H/Q \times G/P$ be a real double flag variety on which H acts diagonally. We are interested in the geometry of X. For example, there are only finitely many H-orbits on X, so that it is of finite type. In this talk, we introduce integral kernel operators which intertwine degenerate principal series of $G = \mathrm{Sp}_{2n} \mathbb{R}$ and $H = \mathrm{GL}_n \mathbb{R}$. For the kernel of the operator, we use relative invariants associated with a prehomogeneous vector space naturally arising from the geometry of X. The integrals have complex parameters, and at certain integral parameters we get finite dimensional representations as images.

Shu-Yen Pan
National Tsing Hua University, Hsinchu

Finite theta correspondence, unipotent support and orbit correspondence

In this talk we show that if two irreducible representations are paired by the theta correspondence for a finite symplectic/orthogonal dual pair, then the unipotent supports of their Alvis-Curtis duals are paired by the orbit correspondence. Moreover, if the orthogonal group in the dual pair is even and two representations are special unipotent, we show that the reverse implication also holds.

Binyong Sun
Zhejiang University, China

Some theorems in the representation theory of classical Lie groups

After introducing some basic notions in the representation theory of classical Lie groups, I will explain three results in this theory: the multiplicity one theorem for classical groups, the conservation relation for local theta correspondence, and the non-vanishing hypothesis at infinity for Rankin-Selberg convolutions. Some relevant or consequent results will also be explained.

Shuichiro Takeda
Osaka University, Japan

The local theta correspondence: an infomercial talk

This is an infomercial talk on the forthcoming book on the local theta correspondence authored by W.T. Gan, S. Kudla, and the speaker. In this talk, I will give an overview of the local theta correspondence, especially focusing on what is in the book.

Chenyan Wu
The University of Melbourne, Australia

Theta correspondence and simple factors of global Arthur parameters

Let π be a cuspidal automorphic representation of a classical group or a metaplectic group. We show an exact relation between two invariants associated to π, one being the lowest occurrence index of π with respect to theta lifts and the other the location of the maximal positive pole of an Eisenstein series attached to π. As an application, we use this relation to show that certain global Arthur packets cannot contain cuspidal automorphic representations.